External Consensus in Multi-agent Systems with Large Consecutive Data Loss under Unreliable Networks

Nurul Adilla Mohd Subha [1, 2*], Guo-Ping Liu [1]

[1] Faculty of Computing, Engineering and Science, University of South Wales, CF37 1DL, UK
[2] Department of Control and Mechatronics Engineering, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Malaysia
[*] nurul.mohdsubha@southwales.ac.uk

Abstract: This paper discusses the external consensus problem for non-identical networked multi-agent systems (NMAS) with network data loss, considering both uniform and random consecutive data losses induced by long periods of transmission failure. A novel approach using a combination of prediction strategy and constant control gain based on gain error ratio (GER) is proposed for solving the problem. The main contribution of this paper is a simple and practical way of selecting control gain using GER to help overcome prediction inaccuracies caused by the prediction process during a long period of consecutive data loss. The feasibility and performance of the proposed consensus protocol is demonstrated through simulation and practical experiment.

1. Introduction

For the last decades, the networked multi-agent system (NMAS) related researches have been noticeably active based on published research findings within the control community. The NMAS structure particularly has spark the curiosity of researchers that interested in finding a reliable control strategy especially for the large scale process control system where it covers a large geographical area, consisting of many subsystems (agents), sensors and actuators. As a large system, performing a cooperative mission among number of agents rather than solo mission in handling the work load theoretically helps to ease the burden in the controller. Completing the task cooperatively is much more feasible and reliable as the work is splits into smaller parts which is much easier to be completed.

Significant advancements in network communication technology and capability have created a promising opportunity for practical application of the NMAS structure. Application of network communication technology in control systems is desirable because it increases system modularity and flexibility and so reduces infrastructure complexity in future system expansion. This, however, has creates a new set of problems for users, such as data delay and data loss, which these problems significantly reduce the system's performance and robustness.

One of the common cooperative problems for NMAS is the consensus problem. Consensus is a type of cooperative control problem in which agents within the system communicate with each other and converge to a specific common value. This value is a target point for all agents to converge upon and may represent physical quantity such as angle, level, velocity, etc. Applications of cooperative control and consensus can be found in networked physical systems such as mobile robots [1], underwater vehicles [2], and district heating systems [3]. These have also been investigated in terms of theoretical development, summarized in [4].

In most existing works, analysis of NMAS consensus is studied in the context of single-integrator dynamics. For example, in [5-8], the authors have provided a significant fundamental work in analysing the NMAS consensus problem. They cover most of the major problems related to NMAS theory and applications, including undirected and directed networks, fixed or switching topology, with and without network delay; however, none of this work has discussed the problem of data loss in depth. Some notable work related to NMAS average consensus with data loss is found in [9]: two types of consensus protocol are discussed and compared by considering data losses within the network happening simultaneously among identical integrator agents. Besides that, work focusing on the data loss problem with single-integrator NMAS is presented in [10]. To extend the complexity of the single-integrator framework, there are many existing papers dealing with ideas related to the application of double-integrator dynamics in NMAS. For instance, [11] considers the leader-follower framework with a data loss scenario where both Lyapunov and linear matrix inequality (LMI) methods are used to derive the sufficient condition for the proposed controller stabilization. By imposing the queuing mechanism, [12] shows that the consensus can be achieved in the presence of random network delay and loss. To emphasize on more complicated agents' dynamics, average consensus of linear NMAS with data loss is investigated in [13] and it has been proved that the mean-square consensus can be achieved if the union graph is connected. In [14], multi-order integrator NMAS with packet loss is studied and has been proved its analysis effectiveness using stochastic and interval matrix theory. The claim of ease of implementation in real applications is, however, not supported by any experimental results. Instead of operating the consensus protocol using time-triggered control, [15] studies event-triggered control for solving the average consensus problem with packet dropout.

Much effort has also been devoted to improving the NMAS consensus performance by maximizing the convergence speed. For example, in [16], the improvement of convergence speed for the average consensus with double-integrator dynamics has been explicitly presented through optimization strategy and analytic solutions. In [17], which also focuses on the convergence speed with the same dynamics, necessary and sufficient conditions for the consensus are derived and the maximum convergence speed is achieved by introducing *gain* that can be determined using the root locus and the second smallest and largest eigenvalues of the Laplacian matrix. In [18], the authors discuss the improvement in convergence speed during occurrence of data loss if second-order neighbour information is utilized in the consensus protocol with an unchanged network topology. Achieving consensus in the shortest convergence time is important for minimizing the effect of data loss.

Note that most of existing studies on the consensus problem with data loss are derived and solved theoretically, with verification only by numerical simulation. Even though the contribution of theoretical studies towards the culmination of the fundamental knowledge related to the consensus problem is undeniably important, most of the results cannot be replicated in real practice [19, 20].

Furthermore, with respect to consensus problem in NMAS application, there are obvious gaps in the research area relating to investigation into data loss and consecutive data losses (CDL) effects on higher order NMAS. Understanding of the problems and development of the solutions for these issues is vital to ensure stable and reliable NMAS performance in practical application.

In order to produce practical solutions to the consensus problem, we present a functional method of external consensus with prediction dealing with maximum allowable CDL within non-identical NMAS. The external consensus protocol is a distributed algorithm that directs all agents in NMAS to converge to an external reference input specified to only one agent (Agent 1 as a leader) via a local communication network. In external consensus protocol, Agent 1 has the ability to transmit and receive information from its neighbouring agent(s). This is the contrasting characteristic of the external consensus protocol compared to the conventional leader following consensus protocol - where the leader agent only has the ability to transmit the information to the other agent(s) in the system.

Previous results in [21] have presented fundamental investigation into practical applications of external consensus with prediction within non-identical NMAS; within the scope of this work, the maximum number of CDL is chosen to be one step higher than the allowable CDL in the consensus problem using the *previous value* method.

With the previous value method, the received data is stored in the buffer and only the latest stored value is applied if the transmission failure occurs. It has been proven successful in improving the performance of consensus when data is lost during certain consecutive sequences in [9]. Thus, the previous value method is taken as a performance benchmark for comparison with the proposed solution, which is expected to be able to deal with higher CDL than is allowable using the previous value method.

The main contribution of this work lies in the application of gain error ratio (GER) within external consensus with prediction. Inspired by [22], GER is a ratio value calculated using the measured output error between agents and their neighbours. The GER is proposed to solve inaccuracies in the prediction when lengthy CDL (long enough to cause instability in NMAS using the previous value method) occurs in NMAS. It acts as a coupling between agents, which helps to minimize the consensus error in order to improve the performance of external consensus with prediction. Furthermore, through simulation and experimental tests, application of GER has also shown capability for minimizing the convergence time of the agents within NMAS.

In this paper, the *network predictive control algorithm with GER* formula (NPCA-GER) is proposed and a performance comparison between the proposed method and the previous value method in solving the external consensus problem is presented. To explore the capability of the GER formula, both uniform and random CDL problems are considered. In addition, to represent a real application scenario, the simulated models of NMAS agents are obtained from empirical data of real test rigs through a system identification process using MATLAB. Experimental tests using the actual test rigs are also performed to validate the simulation results.

The paper is organized into six main sections. In Section 2, the fundamental matrices in consensus problem are introduced. In addition, the common methods in dealing with data loss problem are described and the external consensus protocol with and without data loss compensation are briefly presented. In Section 3, the novel method of combining the prediction strategy with the GER formula is illustrated. In Section 4, the feasibility of the proposed consensus protocol is presented through numerical simulations. In Section 5, numerical simulation results are validated by implementing the proposed work in the practical experiments with two water level-control test rigs. Finally, in Section 6, the conclusion of the work is given.

2. Consensus of Non-identical NMAS with Network Data Loss Problem

2.1 Preliminaries

The data exchange is modelled by a simple undirected graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be an undirected graph of order *n* with a set of nodes or agents $\mathcal{V} = \{v_1, v_2, .., v_n\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge from i to j is denoted by $e = (v_i, v_j)$ to indicate that agent *j* can receive information from agent *i* and vice versa. In an undirected graph, an edge from i to j and j to i has no exact direction and so the graph has the positive unweighted adjacency matrix $a_{ij} = a_{ji} = 1$ for all *i*,*j*. No self-loop is allowed and hence $a_{ii} = a_{jj} = 0$. The set of neighbour agents i is denoted by $N_i = \{j \in \mathcal{V}: (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix L with respect to the undirected graph $\mathcal{G}$ can be simply obtained as

$$L = [l_{ij}]_{n\times n}$$

where

$$l_{ij} = \begin{cases} |N_i|, & if\ i = j \\ -1, & if\ i \neq j\ and\ v_i\ is\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$$

In this work, the Laplacian matrix L has non-zero elements because every agent is interconnected to one another. Consequently, row-sums of L will be zero. Therefore, the zero eigenvalue of L will be the smallest eigenvalue (that is, $\lambda_1 = 0$) if and only if $\mathcal{G}$ has a spanning tree and $\mathcal{G}$ is strongly connected. The second-smallest eigenvalue of L satisfies $\lambda_2 > 0$ if and only if $\mathcal{G}$ is connected [23].

2.2 Network Data Loss Problem

There are three common methods used to solve the data loss problem when the signal transmission fails from agent *j* to agent *i* in NMAS:

i. Method I: Zero input. The output of agent *j* is set to zero, that is, $y_j(k|k) = 0$. This method usually gives poor output results; hence, it will not be further explored in this work.

ii. Method II: The previous value of agent *j* is used. This value represents the latest stored output information of agent *j* at time k, that is, $y_j(k|k) = y_j(k-1)$. This method produces acceptable results for a short CDL or a system with a small sampling time.

iii. Method III: The prediction sequence of agent j is computed at time k and stored at agent i after transmission. During data loss across the network, one predicted value will be chosen from the stored sequence available at agent i, that is, $y_j(k|k) = y_j(k|k-p), p = \{1,2, \dots, p_{max}\}$, depending on the magnitude of CDL. Maximum CDL between NMAS agents is defined as p_{max}.

In this work, NMAS consensus performance using methods II and III is compared through simulation and experimentation. Two types of CDL are considered: uniform CDL (occurring at uniform intervals) and random CDL (occurring at random irregular intervals).

2.3 External Consensus Protocol with Network Data Loss Problem

The external consensus protocol with the effect of network data loss for Method II can be expressed as

$$u_i(k) = G_i(z^{-1})\left[\sum_{i=1}\big(R(k) - y_i(k)\big)\right. \tag{1}$$

$$\left. - \sum_{\substack{i=1 \\ j \in N_i}}^{n} \Big(\big(1-\alpha_{ji}\big)\Big(y_i(k-1) - y_j(k-1)\Big) + \alpha_{ji}\big(y_i(k) - y_j(k)\big)\Big)\right]$$

where $\alpha_{ji} = 1$ if there is no data loss from agent j to agent i and $\alpha_{ji} = 0$ otherwise. In this work, the protocol above is compared with the proposed consensus protocol using Method III, which can be described as follows:

$$u_i(k) = G_i(z^{-1})\left[\sum_{i=1} K_r\big(R(k) - y_i(k)\big) - \sum_{\substack{i=1 \\ j \in N_i}}^{n} K_{ij}\big(y_i(k) - \big(1-\alpha_{ji}\big)y_j(k|k-p) - \alpha_{ji}y_j(k)\big)\right] \tag{2}$$

where K_r and K_{ij} are the calculated constant control gains using the proposed GER formula. Further discussion about GER will be presented in Section 3.4. The external reference input is denoted by $R(k)$, $u_i(k)$ is the control input, $y_i(k)$ and $y_j(k)$ are the measured outputs of agents i and j respectively and n represents the number of NMAS agents. Here, the effects of different maximum numbers of uniform CDL are considered. The number of CDL is denoted by p. During the loss period, the prediction output $y_j(k|k-p)$ that has been stored at agent i will be applied. The occurrence of data loss is assumed to be uniform among agents where all the agents are subject to the same data loss.

Following the consensus protocol in (2), K_r exists between agent 1 and the external reference input only while K_{ij} exists between agent i and its neighbours in NMAS for $i = 1, 2, \dots, n$. The proposed consensus protocol in (2) is said to solve the external consensus problem if and only if $\lim_{k\to\infty}\|R(k) - y_1(k)\| = 0$ and $\lim_{k\to\infty}\|y_i(k) - y_j(k)\| = 0$ if there exist control gains for $i = 1, 2, \dots, n$.

In (1), gain is not introduced because Method II requires the gain to be selected using a trial and error technique which is not easily repeatable in real applications. Moreover, in multi-agent systems, implementation of this technique becomes harder with every additional agent in the system. Even though there have been some previous efforts to try to develop analytical methods for selecting suitable gain for Method II, these efforts are not favourable as they require complex, in-depth mathematical analysis, especially with the involvement of non-identical linear NMAS [24-25].

In (2), gain is introduced into the equation using GER. The application of the prediction algorithm in NMAS conveniently produced a specific error value for each specific sequence of CDL and therefore specific constant gain can be identified for each sequence. This characteristic is the foundation for the application of GER within the system where constants K_r and K_{ij} are utilized throughout the system even with a varied sequence of CDL.

In this work, the acceptable measured experimental output is considered to be within ±5% of its final value or set point. This tolerance band takes into account that the agents' models used in the prediction are best-effort-approximated models of the real test rigs.

3. Network Predictive Control Algorithm

3.1 Agent Model

Consider that the non-identical NMAS can be described by the following model:

$$
\begin{aligned}
y_i(k+1) &= \frac{B_i(z^{-1})}{A_i(z^{-1})}u_i(k) = P_i(z^{-1})u_i(k) \\
A_i(z^{-1}) &= 1 + a_{i1}z^{-1} + \cdots + a_{in_{ai}}z^{-n_{ai}} \\
B_i(z^{-1}) &= b_{i0} + b_{i1}z^{-1} + \cdots + b_{im_{bi}}z^{-m_{bi}}
\end{aligned}
\tag{3}
$$

where, for $i = 1, 2, \dots, n$, $u_i(k)$ is the control input, $y_i(k)$ is the measured agent output, n denotes the number of NMAS agents, n_{ai} is the polynomial order of $A_i(z^{-1})$ and m_{bi} is the polynomial order of $B_i(z^{-1})$ with $n_{ai} \geq m_{bi}$.

The virtual decentralized local controller of non-identical NMAS is designed to achieve the external consensus of NMAS without considering the occurrence of network data loss, and it can be represented as

$$G_{ci}(z^{-1}) = \frac{D_i(z^{-1})}{C_i(z^{-1})}$$
$$C_i(z^{-1}) = 1 + c_{i1}z^{-1} + \cdots + c_{in_{ci}}z^{-n_{ci}} \tag{4}$$
$$D_i(z^{-1}) = d_{i0} + d_{i1}z^{-1} + \cdots + d_{in_{di}}z^{-n_{di}}$$

where n_{ci} is the polynomial order of $C_i(z^{-1})$ and n_{di} is the polynomial order of $D_i(z^{-1})$ with $n_{ci} \geq n_{di}$.

For the simplicity of the consensus analysis, the following assumptions can reasonably be made.

a) Each agent i contains information about agent $j \in i \cup N_i$.
b) Each agent i can receive information from agent $j \in i \cup N_i$.
c) Every network transmission between agents is subject to the same type of loss, i.e. uniform or random CDL.
d) Maximum CDL is known.

3.2 Implementation of Networked Predictive Control Algorithm with Control-Gains-Based GER Formula (NPCA-GER)

According to the proposed consensus protocol, the transmission occurs not only for a single datum but also for a sequence of data. In the sequence data, the output and control input prediction sequence up to the possible maximum CDL p_{max} is available. For $i = 1, 2, \dots, n$, the prediction sequence is computed by agent i before the data is transmitted to other agents. The other agents that receive the prediction sequence will store the sequence data and use it whenever a failure in transmission occurs. The design of the generated prediction sequence algorithm is based on the recursive prediction method. To compute the output prediction sequence up to p_{max}, the prediction computation for the control input is also required. To compensate one loss, the one-step-ahead prediction sequence of agent output at time $k - p$ is constructed as follows:

$$y_i(k-p+1|k-p) = -\sum_{f=1}^{n_{ai}} a_{if} y_i(k-f-p+1) + b_{i0} u_i(k-p|k-p) + \sum_{f=1}^{m_{bi}} b_{if} u_i(k-f-p) \tag{5}$$

where

$$\begin{aligned} u_i(k-p|k-p) &= -\sum_{f=1}^{n_{ci}} c_{if}\, u_i(k-f-p) + K_r \sum_{i=1} D_i(z^{-1}) R(k-p) \\ &\quad - \sum_{f=0}^{n_{di}} d_{if} \left(K_r \sum_{i=1} y_i(k-f-p) + K_{ij} \sum_{\substack{i=1 \\ j\in N_i}}^{n} \left(y_i(k-f-p) - \bar{y}_j(k-f-p) \right) \right) \end{aligned} \tag{6}$$

At agent i, the prediction sequences received from its neighbours are stored as $\left[y_j(k-p|k-p)\; y_j(k-p+1|k-p)\; \ldots y_j(k-p+p_{max}|k-p)\right]^T$ in the buffer in the loss compensator (LC) block. The output of LC during either loss or no-loss is denoted as $\bar{y}_j(k) = y_j(k|k-p)$ for simplicity. Another buffer is positioned prior to the NPCA-GER. Therefore, $\bar{y}_j(k-f-p)$ can be obtained directly from the second buffer. At time k, the predicted value chosen from the LC is applied to the controller of agent *i* to compute the $u_i(k)$ in (2) and is also used for the computation in (6). The external reference input $R(k)$ is only connected to agent 1. Then, to compute the second output prediction for two CDL, one-step-ahead prediction of the control input of agent *i* is needed; this can be expressed as

$$\begin{aligned} u_i(k-p+1|k-p) &= -c_{i1} u_i(k-p|k-p) - \sum_{f=2}^{n_{ci}} c_{if}\, u_i(k-f-p+1) \\ &\quad + K_r \sum_{i=1} D_i(z^{-1}) R(k-p+1) \\ &\quad - d_{io} K_r \sum_{i=1} y_i(k-p+1|k-p) \\ &\quad - d_{io} \left(K_{ij} \sum_{\substack{i=1 \\ j\in N_i}}^{n} \left(y_i(k-p+1|k-p) - \bar{y}_j(k-p+1|k-p) \right) \right) \end{aligned} \tag{7}$$

$$-\sum_{f=1}^{n_{di}} d_{if} K_r \sum_{i=1} y_i(k-f-p+1)$$

$$-\sum_{f=1}^{n_{di}} d_{if} \left(K_{ij} \sum_{\substack{i=1 \\ j\in N_i}}^{n} (\, y_i(k-f-p+1) - \bar{y}_j(k-f-p+1)) \right)$$

In (7), the output prediction $y_i(k-p+1|k-p)$ can be obtained through the computation in equation (5). At agent i, the prediction of $\bar{y}_j(k)$ is also required in order to compute equation (7) successfully. Thus, $\bar{y}_j(k-p+1|k-p)$ can be generated using (8), as follows:

$$\bar{y}_j(k-p+1|k-p) = -\sum_{f=1}^{n_{aj}} a_{jf}\bar{y}_j(k-f-p+1) + b_{j0}\bar{u}_j(k-p|k-p) + \sum_{f=1}^{m_{bj}} b_{jf}\bar{u}_j(k-f-p) \qquad (8)$$

where $\bar{u}_j(k)$ can also be obtained from the buffer. The same computation in (6) is made for $\bar{u}_j(k-p|k-p)$, inverting the notation of i and j.

After computation (8), computations (5)–(8) are repeated for two-step-ahead prediction until the possible maximum CDL p_{max} is obtained. Thus, the prediction sequence of agent i from time $k-p+1$ to k for $l_{cs} = 1, 2, \dots, p_{max}$ can be summarized in terms of the general equation in (9). From (9), it can be seen that both the prediction sequence signal and the current signal available at time $k-p$ are required for the computation to be successful.

$$\begin{aligned} y_i(k-p+l_{cs}|k-p) = & -\sum_{f=1}^{min\{n_{ai},l_{cs}-1\}} a_{if} y_i(k-f-p+l_{cs}|k-p) - \sum_{f=l_{cs}}^{n_{ai}} a_{if} y_i(k-f-p+l_{cs}) \\ & + \sum_{f=0}^{min\{m_{bi},l_{cs}-2\}} b_{if} u_i(k-f-1-p+l_{cs}|k-p) \\ & + \sum_{f=l_{cs}-1}^{m_{bi}} b_{if} u_i(k-f-1-p+l_{cs}) \end{aligned} \qquad (9)$$

$$
\begin{aligned}
u_i(k-p+l_{cs}|k-p) = & - \sum_{f=1}^{min\{n_{ci},l_{cs}-1\}} c_{if} u_i(k-f-p+l_{cs}|k-p) - \sum_{f=l_{cs}}^{n_{ci}} c_{if}\, u_i(k-f-p+l_{cs}) \\
& + K_r \sum_{i=1} D_i(z^{-1}) R(k-p+l_{cs}) \\
& - \sum_{f=0}^{min\{n_{di},l_{cs}-1\}} d_{if} K_r \sum_{i=1} y_i(k-f-p+l_{cs}|k-p) \\
& - \sum_{f=0}^{min\{n_{di},l_{cs}-1\}} d_{if} \left(K_{ij} \sum_{\substack{i=1 \\ j\neq i}}^{n} (y_i(k-f-p+l_{cs}|k-p) \right. \\
& \left. \qquad - \bar{y}_j(k-f-p+l_{cs}|k-p)) \right) \\
& - \sum_{f=l_{cs}}^{n_{di}} d_{if} K_r \sum_{i=1} y_i(k-f-p+l_{cs}) \\
& - \sum_{f=l_{cs}}^{n_{di}} d_{if} \left(K_{ij} \sum_{\substack{i=1 \\ j\neq i}}^{n} (y_i(k-f-p+l_{cs}) - \bar{y}_j(k-f-p+l_{cs})) \right)
\end{aligned}
$$

3.3 Data Loss Simulation Model

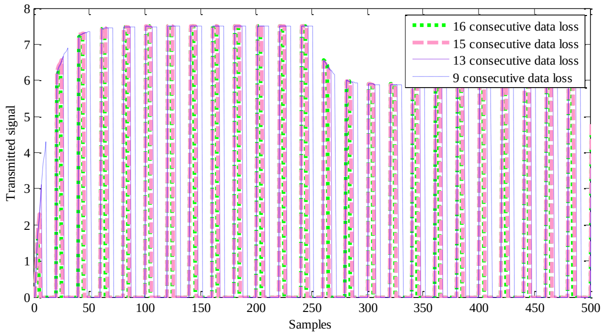
The occurrence of data loss for uniform and random CDL is represented by a data loss simulation model which can be expressed as follows:

$$Uniform\ CDL;\ Out = \begin{cases} 0\ if\ r_s\ > threshold\ value \\ In\ if\ r_s\ \leq threshold\ value \end{cases}$$

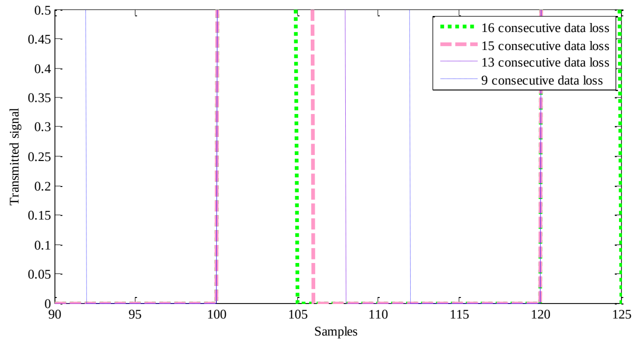
$$Random\ CDL;\ Outrand = \begin{cases} In1\ if\ r_s\ > threshold\ value = 0 \\ \ 0\ if\ r_s\ \leq threshold\ value = 0 \end{cases}$$

where *In* and *In*1 are the inputs to the data loss simulation model which is the prediction sequence of agent *j*, *Out* and *Outrand* are the outputs of the model, and r_s is a set of repeating sequences for uniform CDL, while in random CDL, r_s is a signal builder, and *threshold value* is the pre-set value. The switch acts as a network transmission line to replicate the scenarios of the network operating with and without data loss. The switch propagates one of two inputs (either loss (0) or no-loss (*In* or *In*1)) triggered by the value of the control input r_s . The pre-set value is the value of the control input r_s at which the switch flips to its other input.

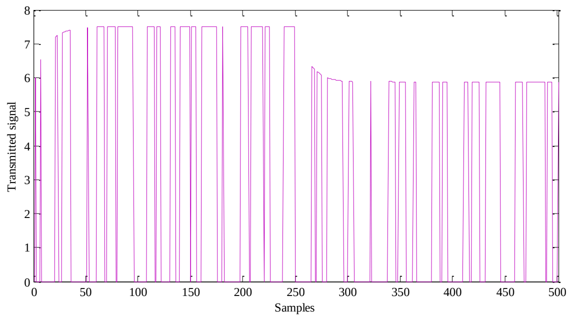
In this work, a signal builder block is used to generate the custom random CDL with maximum CDL set at 16. Maximum CDL is set at 16, based on the performed simulation result which showed that (1) failed to solve the external consensus problem at this level of CDL. The actual simulation result of (1) is presented in Section 4.2. For uniform data loss sequences, four different values of uniform CDL (9, 13, 15 and 16) are considered. The transmitted signals with uniform and random CDL are represented in Fig. 1.



a



b



c

Fig. 1. *Transmitted signal*
a Transmitted signal with different maximum uniform CDL
b Close-up of Fig. 1a
c Transmitted signal with random CDL for $p_{max} = 16$

3.4 Gain Error Ratio (GER) Formula

The development of GER formula is inspired by [22], which can be summarized in the scope of our work as follows:

Table 1 Gain error ratio (GER)

<table>
<tr><th colspan="3">Agent i</th><th>$i = 1$</th><th>$i = 2$</th><th>...</th><th>$i = n$</th></tr>
<tr><td rowspan="5">Control gains required by each agent</td><td colspan="2">Connection between Agent 1 and external reference input</td><td>K_r</td><td></td><td></td><td></td></tr>
<tr><td rowspan="4">Connection between agent i and its neighbours (K_{ij} , $j \in N_i$)</td><td>$j = 1$</td><td></td><td>K_{21}</td><td>...</td><td>K_{n1}</td></tr>
<tr><td>$j = 2$</td><td>K_{12}</td><td></td><td>...</td><td>K_{n2}</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td><td>$\vdots$</td><td></td><td>$\vdots$</td></tr>
<tr><td>$j = n$</td><td>K_{1n}</td><td>K_{2n}</td><td>...</td><td></td></tr>
<tr><td colspan="3">Sum of control gains for agent i</td><td>1</td><td>1</td><td>...</td><td>1</td></tr>
</table>

From Table 1, it can be seen that the GER-proposed control gains for each agent sum to 1.

Consensus output results with prediction without control gains will yield an error for every cycle. The term 'cycle' refers to a period of complete series of data exchanges containing both loss and no-loss periods. The term 'no-loss' refers to a period of successfully transmitted data. The output error between agents may vary.

Mathematically, GER is defined as the ratio of the sum of errors between an agent and each of its neighbouring agents to the sum of errors between an agent and all of its neighbouring agents at time *k*. Control gains K_r and K_{ij} can be formulated as

$$K_r = \frac{\sum_{k=1st\ sample}^{end\ sample} |r(k) - y_1(k)|_{1\ cycle}}{\left(\sum_{k=1st\ sample}^{end\ sample} |r(k) - y_1(k)|_{1\ cycle} + \sum_{\substack{j=2 \\ j \epsilon N_1}}^{n} \sum_{k=1st\ sample}^{end\ sample} \left|y_1(k) - y_j(k)\right|_{no\ loss}\right)} \tag{10}$$

$$K_{ij} = \frac{\sum_{k=1st\ sample}^{end\ sample} \left|y_i(k) - y_j(k)\right|_{no\ loss}}{\left(\sum_{i=1} \sum_{k=1st\ sample}^{end\ sample} |r(k) - y_i(k)|_{1\ cycle} + \sum_{\substack{i=1 \\ j \epsilon N_i}}^{n} \sum_{k=1st\ sample}^{end\ sample} \left|y_i(k) - y_j(k)\right|_{no\ loss}\right)}$$

where *k* refers to the *k*th sample. These error values can be obtained using the values of $r(k)$, $y_i(k)$, and $y_j(k)$ for $i = 1, 2, \ldots, n$ and $j \in i \cup N_i$ from the NMAS consensus result with prediction without control gain through simulations. The result shows a repetitive pattern for every cycle (loss + no-loss) during its steady-state condition. Thus, to simplify this method, any one cycle during the steady-state period can be chosen to calculate K_r and K_{ij}. For example, referring to Fig. 1b, samples 101 to 120 are taken as one cycle during the steady-state period.

Within these 20 samples (1 cycle), there are loss and no-loss situations at different samples. For 15 CDL, a no-loss situation occurs at samples 101 to 105 and a loss situation occurs at samples 106 to 120. Thus, the values of $r(k)$, $y_i(k)$ and $y_j(k)$ at a specified sample are used to calculated the error ratio as follows:

$$K_r = \frac{\sum_{k=101}^{120}|r(k) - y_1(k)|}{\sum_{k=101}^{120}|r(k) - y_1(k)| + \sum_{j\in N_1}^{n}\sum_{k=101}^{105}|y_1(k) - y_j(k)|}$$

$$K_{ij} = \frac{\sum_{k=101}^{105}|y_i(k) - y_j(k)|}{(\sum_{i=1}\sum_{k=101}^{120}|r(k) - y_i(k)| + \sum_{j\in N_i}^{n}\sum_{k=101}^{105}|y_i(k) - y_j(k)|}$$

During losses, only the stored (prediction) value is used and thus the actual error value is not available. However, since the connection between the external reference input $r(k)$ and agent 1 is not subject to loss, the error of the whole cycle is considered. For random CDL, the control gains calculated in (10) can be applied as long as the maximum value of CDL is known.

3.5 NMAS Stability Analysis

Using the results in [21], the matrix (11) is obtained. The closed-loop system is stable if and only if all the eigenvalues of (11) with the corresponding control gains shown below are within the unit circle:

$$\begin{bmatrix} Y_{T(k+1)} \\ U_{T(k)} \end{bmatrix} = \Lambda(p_{max}) \begin{bmatrix} Y_{T(k)} \\ U_{T(k-1)} \end{bmatrix} \tag{11}$$

where

$$\Lambda(p) = \begin{bmatrix} A_T + B_T T & B_T(C_T + W) \\ T & C_T + W \end{bmatrix} \epsilon R^{(\bar{n}+\bar{m}+2)n \times (\bar{n}+\bar{m}+2)n}$$

$$U_{T(k-1)} \triangleq [u_1(k-1) \ \dots \ u_1(k-\bar{m}_1-1) \ u_2(k-1) \ \dots \ u_n(k-\bar{m}_n-1) \ ]^T \in R^{n(\bar{m}+1)}$$

$$Y_{T(k-1)} \triangleq [y_1(k-1) \ \dots \ y_1(k-\bar{n}_1-1) \ y_2(k-1) \ \dots \ y_n(k-\bar{n}_n-1) \ ]^T \in R^{n(\bar{n}+1)}$$

$$A_T \triangleq diag\{A_1, A_2, \dots, A_n\}$$

$$B_T \triangleq diag\{B_1, B_2, \dots, B_n\}$$

$$C_T \triangleq diag\{C_1, C_2, \dots, C_n\}$$

and $l_{cs} = 1, 2, \dots, p_{max}$. For $i = 1, 2, \dots., n$, $\bar{n}_i$ represents the maximum value of n_{di} and n_{ai}, while $\bar{m}_i$ represents the maximum value of n_{ci} and m_{bi}. The polynomial order of agent i and its virtual local controller is considered to be equal so that $\bar{n}_i = \bar{n}$ and $\bar{m}_i = \bar{m}$. A detailed definition of $A_T, B_T, C_T, T,$ and W can be found in [21], where detailed analysis is presented. Therefore, the overall closed-loop NMAS with control-gains-based GER formula is stable for reaching the external consensus if this criterion is fulfilled.

4. Simulation Results

4.1 Simulation Models

In this section, a numerical example is given to illustrate the effectiveness of the proposed formula. Consider NMAS in a fixed topology with n = 2 indexed by 1 and 2. The dynamics of agent i (i = 1, 2) are described by the system model (3), where

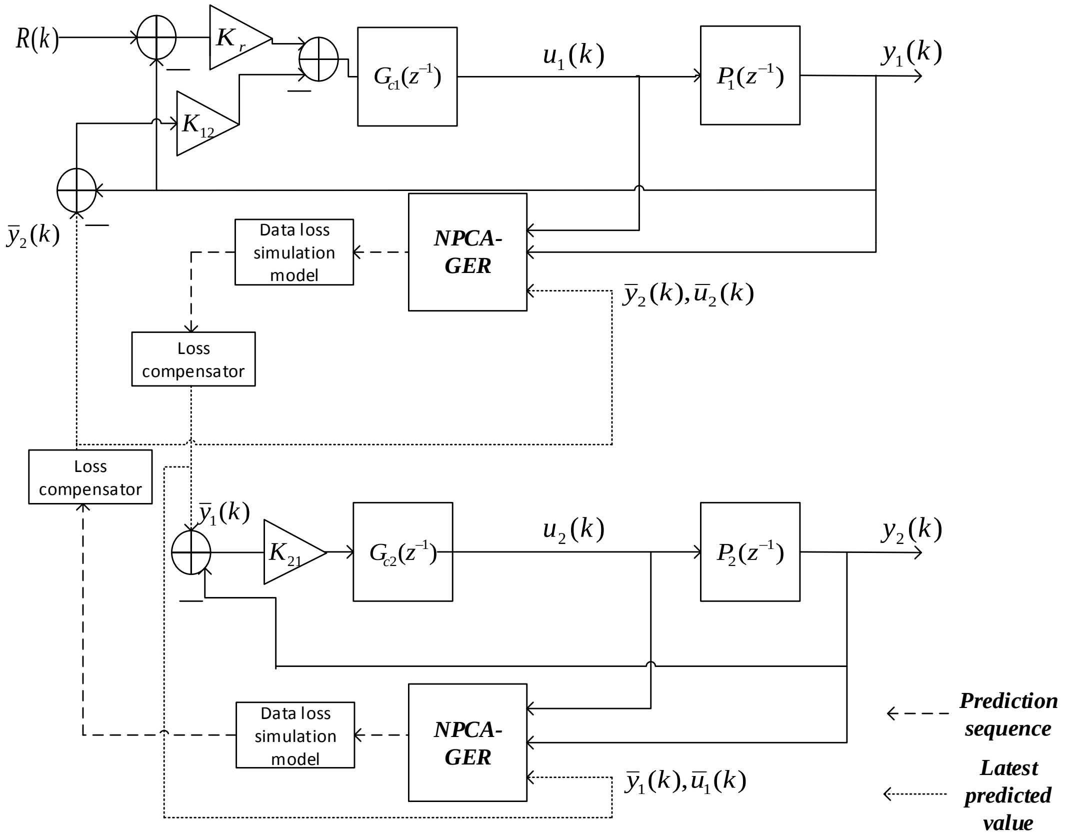
$$\begin{aligned} P_1(z^{-1}) &= \frac{0.06354z^{-1} + 0.00497z^{-2}}{1 - 0.9692z^{-1}} \\ P_2(z^{-1}) &= \frac{0.08345z^{-1} + 0.0222z^{-2}}{1 - 0.7184z^{-1} + 0.01505z^{-2}} \end{aligned} \tag{12}$$

The model of $P_i(z^{-1})$ is an approximated model of the real test rigs. The transfer functions of virtual local controllers for both agents are obtained by employing the Proportional-Integral (PI) controller prior to NPCA-GER design to ensure that the closed-loop agent's system without data loss is stable. The transfer functions are

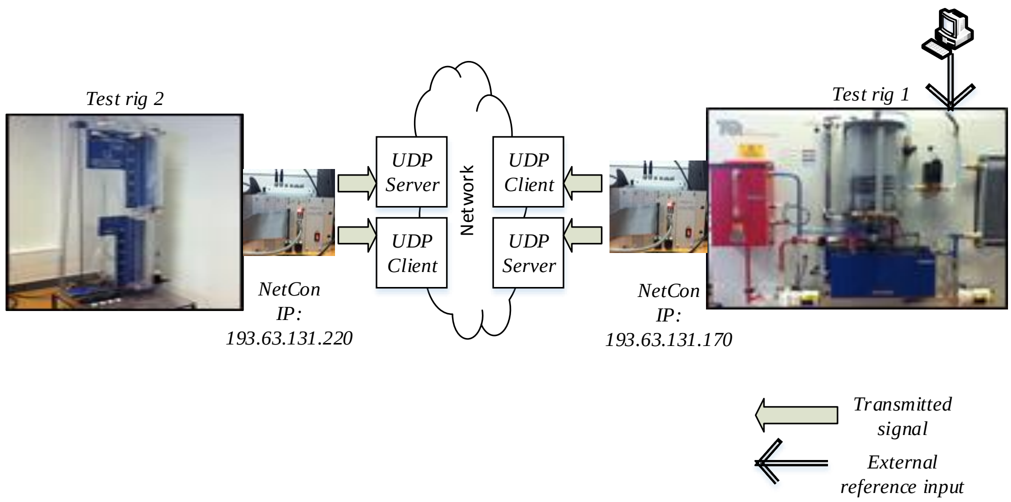
$$\begin{aligned} G_{c1}(z^{-1}) &= \frac{1.35 - 1.31z^{-1}}{1 - z^{-1}} \\ G_{c2}(z^{-1}) &= \frac{1 - 0.6z^{-1}}{1 - z^{-1}} \end{aligned} \tag{13}$$

4.2 NMAS Consensus

In this section, a simulation study for NMAS consensus with uniform and random CDL is considered. NMAS with n = 2 is presented to demonstrate the effectiveness of the proposed consensus protocol. The simulation diagram is shown in Fig. 2a.



a



b

Fig. 2. *Block diagram of NMAS*
a Simulation
b Practical experiment

The NMAS can be represented by two non-identical $P_i(z^{-1})$ and the corresponding NPCA-GER. The network is represented by the data loss simulation model described in Section 3.3. The simulation was carried out and 500 sample output values were recorded at an interval period of 1 s. The comparison is made between (1) and (2) for uniform and random CDL, is illustrated in Figs. 3 to 5.

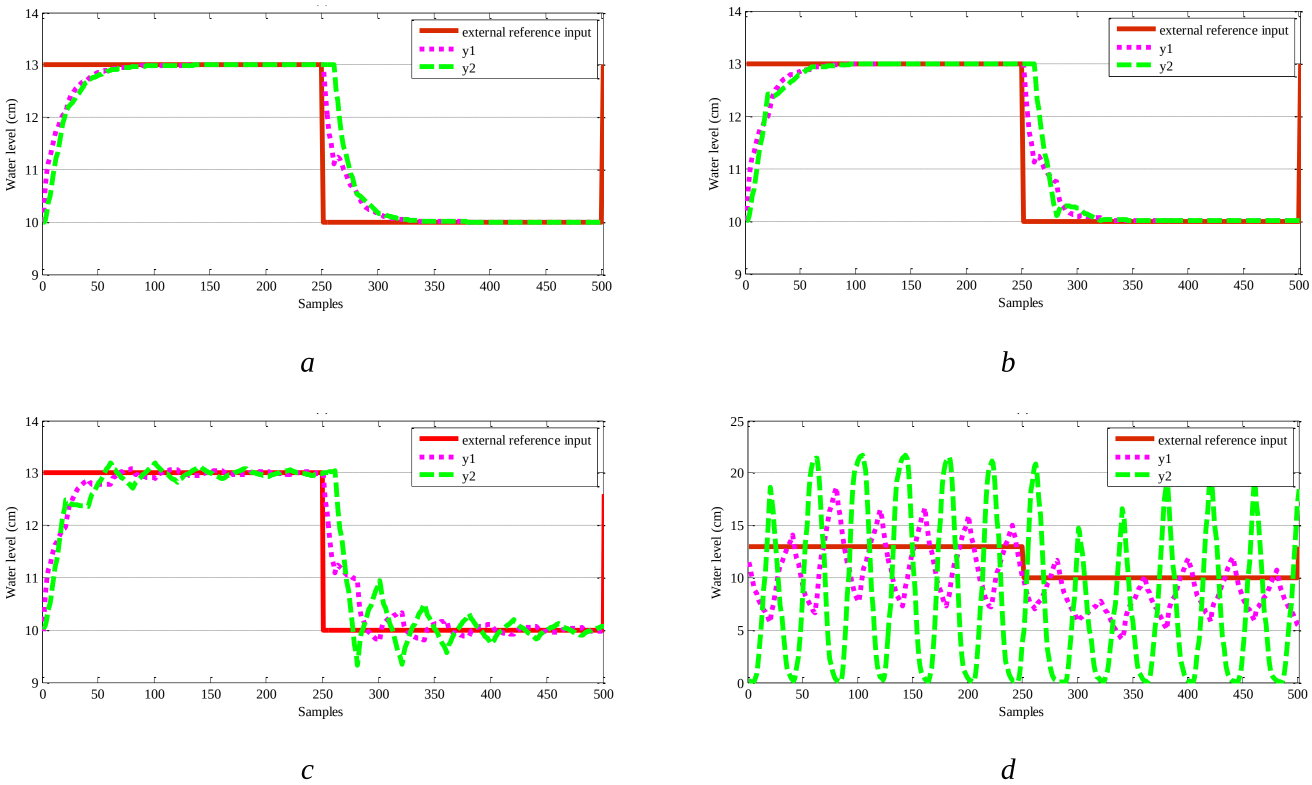
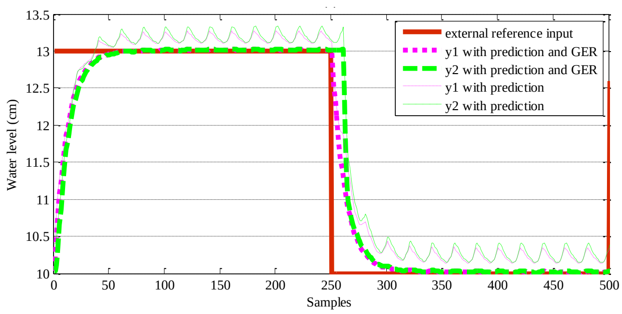
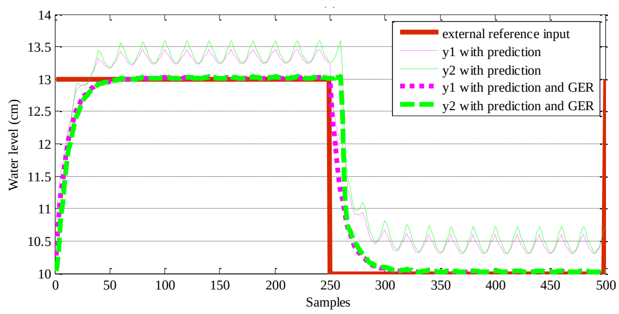


Fig. 3. *External consensus results for* (1) *with uniform CDL*
a 9 CDL
b 13 CDL
c 15 CDL
d 16 CDL

Referring to Fig. 3, it can be seen that by employing (1), the external consensus problem can only be solved up to 15 CDL. Furthermore, the convergence time quadruples from 70 s up to 150 s when the number of CDL is increased from 9 to 15. The consensus results with NPCA-GER in (2) are illustrated in Fig. 4.



a



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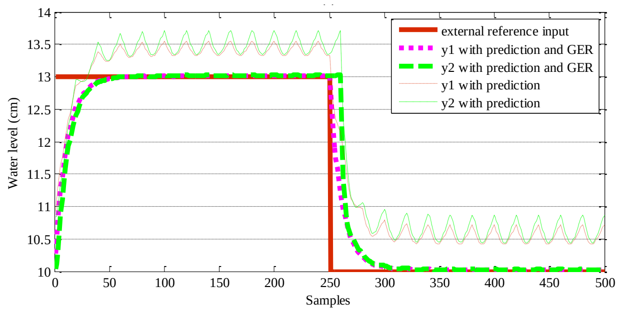
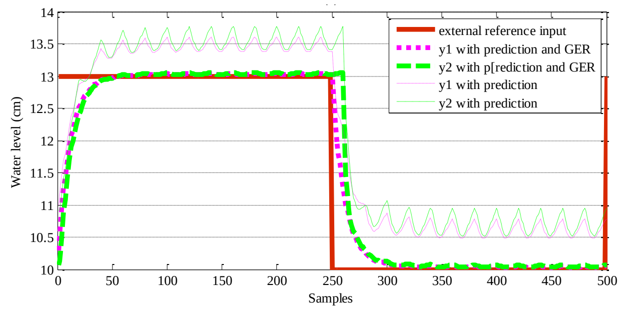


c



d

Fig. 4. *External consensus results by using NPCA-GER in* (2) *(thick line) with uniform CDL*
a 9 CDL
b 13 CDL
c 15 CDL
d 16 CDL

From Fig. 4, it is observed that the graph with a thin line shows that the consensus results with prediction and without GER are stable but significant offsets and uniform ripples are present. Output from agent 2 follows the output of agent 1 in the same manner. Furthermore, with an increase in maximum CDL, the error between the external reference input and the output of agent 1 is also increased. Thus, external consensus cannot be achieved. This problem is caused by the prediction error, which increases when the prediction step is increased.

As proposed in Section 3.4, the control gains K_r and K_{ij} are calculated using GER formula in (10) for maximum CDL of 9, 13, 15, and 16. The calculated values of K_r and K_{ij} are shown in Table 2.

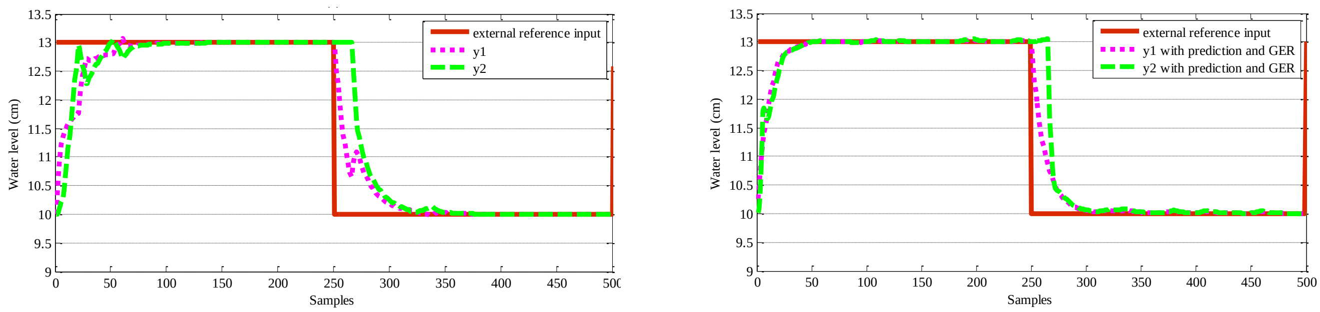
Table 2 Calculated control gains $\boldsymbol{K_r}$ and $\boldsymbol{K_{ij}}$ using GER formula

Maximum CDL	K_r	K_{12}	K_{21}
9	0.922	0.078	1
13	0.950	0.050	1
15	0.966	0.034	1
16	0.941	0.059	1

By employing the calculated control gains in (2), the external consensus problem is solved as shown in Fig. 4. The obvious improvements can be seen for every level of maximum CDL with an implementation of the GER formula in the prediction strategy. In addition, with NPCA-GER, the consensus problem for NMAS with 16 CDL is successfully resolved.

Another significant advantage resulting from application of NPCA-GER is that the convergence time has not increased with the increase in the number of CDL. It is maintained at approximately 50 s for any number of CDL. This capability enhancement further increases NMAS performance and also substantially strengthens the system robustness in practical applications.

However, for random CDL situations, (1) can still give satisfactory results even though 16 CDL have occurred within the 500 samples frame as shown in Fig. 5a. It must be noted that the random CDL result presented here is not conclusive as it is highly dependent on the characteristic of the introduced data loss sequences. For NPCA-GER, since the maximum tested CDL in Fig. 1c is known to be 16, the previous calculated gains at 16 CDL from Table 2, in which $K_r = 0.941$, $K_{12} = 0.059$ and $K_{21} = 1$, are used to handle this situation. As shown in Fig. 5b, the NMAS with random CDL problem is successfully solved with the convergence time at approximately 50 s, which is similar to the results obtained in the simulation with uniform CDL in Fig. 4.



a *b*

Fig. 5. *External consensus results with random CDL*
a Using (1)
b Using (2)

The stability analysis has been derived for four different number of uniform CDL i.e. $p_{max} = 9, 13, 15, 16$ CDL using models in (12) and (13) and the designed coupling gains in Table 2. The results shows that the eigenvalues for matrix (11) are within the unit circle for the specified number of CDL. Therefore, from this analysis, it can be conclude that NMAS is stable for 9, 13, 15 and 16 CDL with the designed coupling gains in Table 2.

In summary, with the introduction of the NPCA-GER, performance of the NMAS consensus increases and it is capable of solving a higher number of CDL. Furthermore, the system is also capable of solving any number of CDL by using GER gains of the highest number of CDL known within the system.

This characteristic helps to simplify the use of GER in real applications where the number of CDL is not always uniform. Besides that, another significant advantage of using NPCA-GER is that it also increases the performance of the system in terms of convergence time, as can be seen in Table 3.

Table 3 Convergence time comparison (simulation)

CDL	Convergence time			
	Method II (1)		NPCA-GER (2)	
	$R(k) = 13$	$R(k) = 10$	$R(k) = 13$	$R(k) = 10$
9	70s	325s	50s	300s
13	70s	325s	50s	300s
15	150s	470s	50s	300s
16	unreachable		50s	300s
Random	75s	325s	50s	300s

The simulation shows that the system produces low and consistent convergence times for any allowable CDL. To verify these results, we conducted a practical experiment which is explained in the next section.

5. Practical Implementation

5.1 Experimental Setup for NMAS Test Rigs

To further validate the simulation result and the effectiveness of the proposed method, NMAS with two different water level process control test rigs have been built; these are illustrated in Fig. 2b. In the experimental setup, Agent 1 and Agent 2 are represented by Test rig 1 and Test rig 2, respectively. Test rig 1 and Test rig 2 are connected to their own networked controller (NetCon) hardware individually with specified IP addresses.

In this work, Test rig 1 is chosen to be connected directly with an external reference input. The consensus target point is chosen to be within a suitable range that can be achieved by both test rigs. The same prediction algorithm as used in the simulation with calculated GER values of control gains in Table 2 is applied to both test rigs.

5.2 Experimental Results

Referring to the setup in Fig. 2b, 500 samples were collected from both test rigs. Results for (1) and (2) are shown in Figs. 6 to 8 for uniform and random CDL.

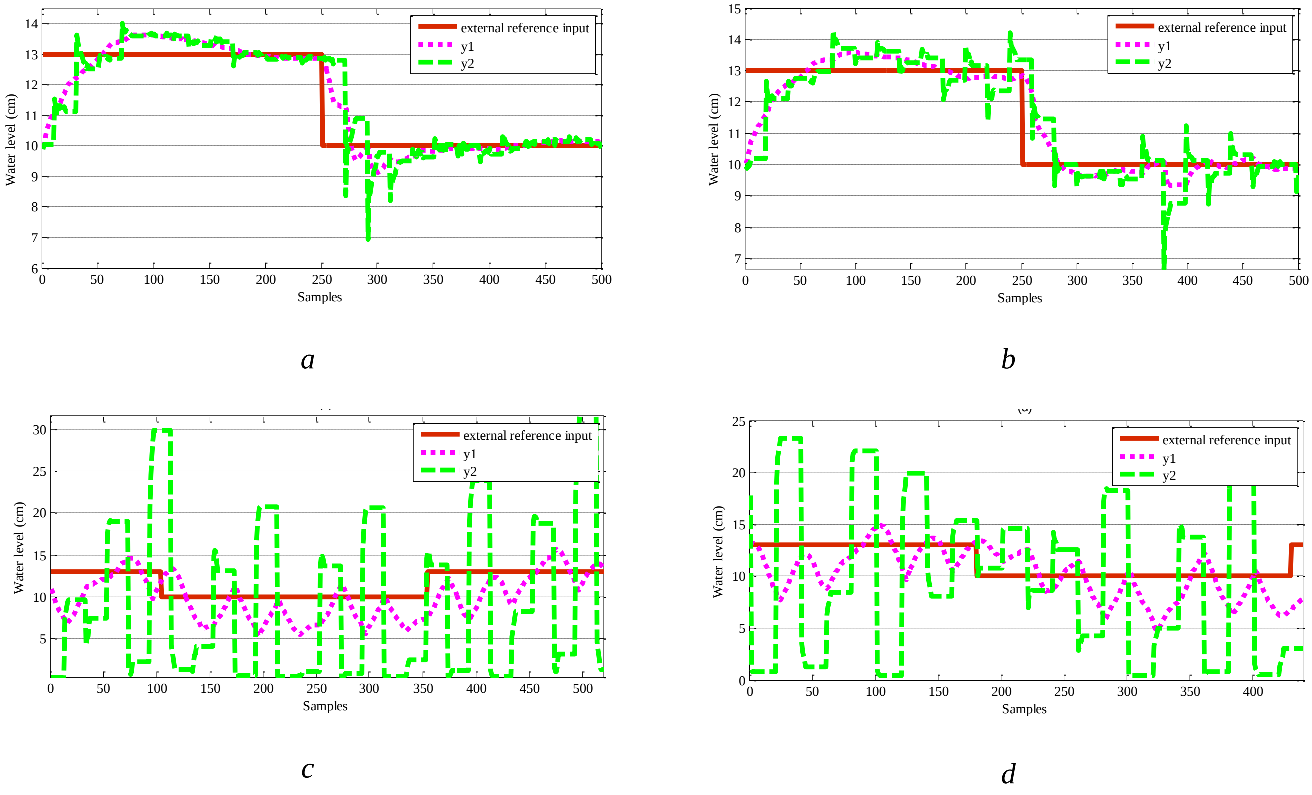
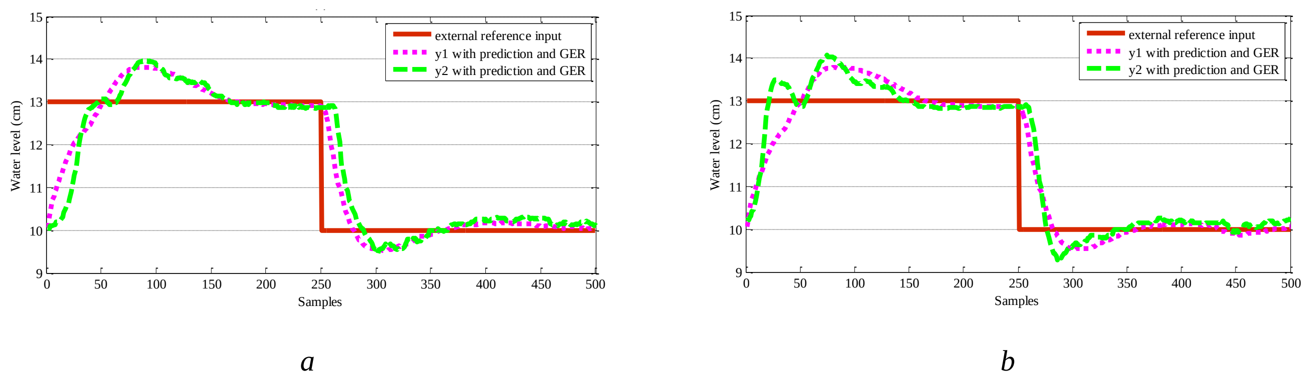
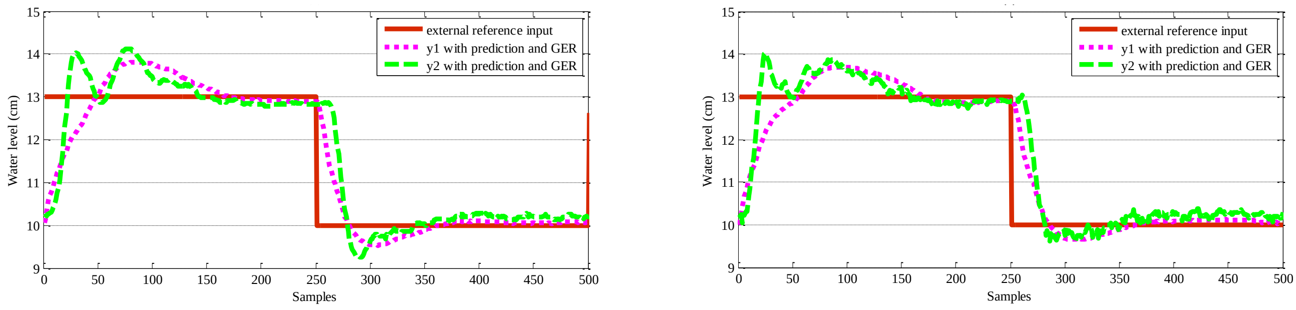


***Fig. 6.** External consensus experimental results for* (1) *with uniform CDL*
a 9 CDL
b 13 CDL
c 15 CDL
d 16 CDL

From Fig. 6, it can be seen that the consensus performance for NMAS on test rigs with (1) is much worse than the results obtained in the simulations. NMAS agents managed to converge only for up to 9 uniform CDL, with significant increase in convergence time. Higher numbers of CDL caused the system to be unstable and fail to reach the consensus. The experimental results for NPCA-GER in (2) are illustrated in Fig. 7.



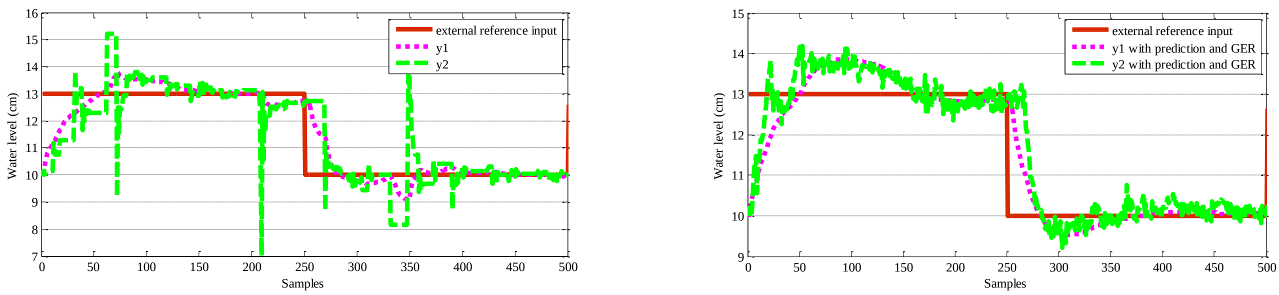


c *d*

Fig. 7. *External consensus experimental results for* (2) *with uniform CDL*
a 9 CDL
b 13 CDL
c 15 CDL
d 16 CDL

From Fig. 7, it can be observed that the NMAS using NPCA-GER achieved its external consensus within its specified tolerance of ±5% from its target point for all levels of CDL sequences. In addition, the results show that with an increment in the number of CDL, the convergence time of NMAS with NPCA-GER does not change, as predicted in the simulations.

In random CDL, (1) produces inconsistent and unsmooth consensus results, as shown in Fig. 8a. However, the proposed solution has produced satisfactory results for random CDL with a similar convergence time, as illustrated in Fig. 8b.



a *b*

Fig. 8. *External consensus experimental results with random CDL*
a Using (1)
b Using (2)

Table 4 Convergence time comparison (experiment)

CDL	Convergence time			
	Method II (1)		NPCA-GER (2)	
	$R(k) = 13$	$R(k) = 10$	$R(k) = 13$	$R(k) = 10$
9	180s	350s	160s	340s
13	unreachable		160s	340s
15	unreachable		160s	340s
16	unreachable		160s	340s
Random	210s	400s	160s	340s

From the results obtained in the experiments, desirable performance is successfully achieved with an application of the proposed NPCA-GER (2) in NMAS. Consensus is achieved for all tested values within maximum allowable data loss. More importantly, the result has shown that convergence time for the NMAS with NPCA-GER is not affected by the number of CDL, as shown in Table 4. The simplicity of the NPCA-GER application, combined with the desirable performance characteristics, makes NMAS with NPCA-GER a robust system which is suitable for practical applications.

6. Conclusion

This paper investigated the external consensus in non-identical NMAS with large CDL. With higher numbers of CDL, the performance of the system deteriorated significantly. This work proposed a novel method to reduce inaccuracies in the prediction process for NMAS to reach external consensus. A combined prediction strategy with a simple formula of choosing the appropriate control gains based on the gain error ratio (NPCA-GER) was presented. Various allowable maximum CDL were tested to verify the effectiveness of the proposed method. The NPCA-GER was verified not only in numerical simulation but also by practical experiment with two non-identical water level control test rigs under intranet connection (university network). The proposed method was not only successful in solving the external consensus problem but also improved the convergence time for non-identical NMAS with data loss. These results show that the proposed NPCA-GER application within NMAS is a feasible solution and a practically capable option for solving external consensus NMAS problems.

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